

## On Duality Principle in Exponentially Lévy Market

Olabisi O. Ugbebor<sup>1</sup> and Sunday Onos Edeki<sup>+2</sup>

---

### Abstract

This paper describes the effect of duality principle in option pricing driven by exponentially Lévy market model. This model is basically incomplete - that is; perfect replications or hedging strategies do not exist for all relevant contingent claims and we use the duality principle to show the coincidence of the associated underlying asset price process with its corresponding dual process.

The condition for the ‘unboundedness’ of the underlying asset price process and that of its dual is also established. The results are not only important in Financial Engineering but also from mathematical point of view.

**Mathematics Subject Classification:** 60G51, 62P05, 91B25, 91B26

**Keywords:** Option pricing, Lévy processes, incomplete market, option dual and primal.

### References

- [1] D. Applebaum, *Levy Processes and Calculus*, Second edition, Cambridge University Press, 2009.
- [2] L. Bachelier, *Théorie de la Spéculation*. Paris, Gauthier-villars, 1900. Translated in Cootner 1964.
- [3] D. S. Bates, The Skewness Premium: Option Pricing Under Asymmetric Process. *Advances in Futures and Options Research, Elsevier* **9**, (1997), 51-82.

---

<sup>1</sup> Mathematics Department, University of Ibadan, Oyo State Nigeria. E-mail: [ugbebor1@yahoo.com](mailto:ugbebor1@yahoo.com)

<sup>2</sup> Mathematics Department, University of Ibadan, Oyo State Nigeria. E-mail: [soedeki@yahoo.com](mailto:soedeki@yahoo.com)

<sup>+</sup> Corresponding Author

- [4] F. Black, and M. Scholes, The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, **81**(3), (1973), 637-654.
- [5] F. Bolshuizen, A. W. van der Vaar, H. van Zanteen et. al., *Stochastic Processes for Finance-Risk Management Tools*. [www.math.sc.edu/Bolshuizen/](http://www.math.sc.edu/Bolshuizen/), 2006.
- [6] E. Eberlein, and K. Prause, *The General Hyperbolic Model: Financial Derivatives and Risk Measures*, FDM preprint 56, University of Freiburg, 1998.
- [7] E. Eberlein, A. Papapantoleon and A. N. Shirryaev, *Esscher Transform and Duality Principle for Multidimensional Semimartingale*, [www.math.sc.edu/EPS/](http://www.math.sc.edu/EPS/), 2009.
- [8] S. O. Edeki, *Applications of Lévy Processes in Finance- duality principle approach*, Unpublished, Msc Thesis, Mathematics Department, University of Ibadan, Nigeria, 2010.
- [9] J. Fajardo, and E. Mordecki, Skewness Premium with Lévy Processes, *Working Paper*, IBMEC, (2006).
- [10] J. Fajardo, and E. Mordecki, Symmetry and Duality in Levy Markets, *Quant. Finance*, **6**, (2006), 219-227.
- [11] A. Friedman, *Stochastic Differential Equations and Applications*, Academic Press, New York, San Francisco, London, 1975.
- [12] J. M. Harrison and S. R. Pliska, Martingales and Stochastic Integrals in the Theory of Continuous Trading. *Stochastic Processes and their Applications*, **11** (3), (1981), 215-260.
- [13] R. V. Ivanov, On the Pricing of American Options in Exponentially Lévy Markets. *J. Applied Prob.* **44**, (2007), 407-414.
- [14] J. Kallsen and A. N. Shiryaev, The Cumulant Process and Esscher's Change of Measure, *Finance and Statistics*, **6** (4), (2002), 397-428.
- [15] P. E. Kloeden and E. Platen, *Numerical Solutions of Stochastic Differential Equations*, Springer-Verlag, 1992.
- [16] B. K. Leonid, and G. Yakov, *Theory of Probability and Random Processes*, Second edition Springer-Verlag, New York, 2007.
- [17] D. B. Madan, and E. Seneta, Chebyshev Polynomial Approximations and Characteristic Function Estimation, *Journal of the Royal Statistical Society Series B*, **49** (2), (1987), 163-169.
- [18] B. Mandelbrot, The Variation of certain speculative prices, *Journal of Business*, **36**, (1963), 394-169.
- [19] K. Masaki, *Stochastic Processes with Applications to Finance*, [www.math./Masaki/](http://www.math./Masaki/), 2001.

- [20] W. Margrabe, The Value of an Option to exchange one asset for another. *Journal of Finance*, **33**, (1978), 177-186.
- [21] Y. Miyahara, *A Note on Esscher Transformed Martingale Measures for Geometric Lévy processes*. E-mail: y-miya@econ.nagoya-cu.ac.jp, 2004.
- [22] D. Nualart, and W. Schoutens, *BSDE's and Feynman-Kac Formula for Lévy Processes with Applications in Finance*. www.math.sc.edu, 2001.
- [23] B. Oksanda, *Stochastic Differential Equations: An introduction with Applications*. Sixth edition, Springer-Verlag, 2000.
- [24] M. F. M, Osborne, Rational Theory of Warrant pricing. *Industrial Management review*, **6**, (1965), 13-32.
- [25] A. Papapantoleon, *Applications of semimartingales and Lévy processes in Finance-duality and valuation*. PhD Thesis, Univ. Freiburg, 2007.
- [26] K. Prause, *The Generalized Hyperbolic Model: Estimation, Financial Derivatives, and Risk Measures*. Dissertation. Mathematische Fakultät der Albert-Ludwigs-Universität Freiburg im Breisgau, 1999.
- [27] S. Raible, *Lévy processes in Finance- Theory, Numerics, and Emperical Facts*, www.math.sc.edu/Raible/ , 2000.
- [28] A. A. F. Saib and M. Bhuruth, *Option Pricing of jump Diffusion Models under Exponential Lévy models using Mathematica*, 2008.
- [29] P. Samuelson, Economics Theory and Mathematics – An appraisal, Cowles Foundation Paper 61, Reprinted from *American Economic Review*, **42**, (1952), 56-69.
- [30] A. N. Shiryaev, *Essentials of Stochastic Finance – Facts, Models, Theory*, World Scientific, 1999.